

# Vacuum polarization by a composite topological defect

E. R. Bezerra de Mello<sup>1\*</sup> and A. A. Saharian<sup>1,2†</sup>

<sup>1</sup>*Departamento de Física-CCEN, Universidade Federal da Paraíba  
58.059-970, Caixa Postal 5.008, João Pessoa, PB, Brazil*

<sup>2</sup>*Department of Physics, Yerevan State University,  
375025 Yerevan, Armenia*

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## Abstract

In this paper we analyze one-loop quantum effects of a scalar field induced by a composite topological defect consisting a cosmic string on a  $p$ -dimensional brane and a  $(m+1)$ -dimensional global monopole in the transverse extra dimensions. The corresponding Green function is presented as a sum of two terms. The first one corresponds to the bulk where the cosmic string is absent and the second one is induced by the presence of the string. For the points away from the cores of the topological defects the latter is finite in the coincidence limit and is used for the evaluation of the vacuum expectation values of the field square and energy-momentum tensor.

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## 1 Introduction

In recent years the braneworld model has received renewed interest. By this scenario our world is represented by a sub-manifold, a three-brane, embedded in a higher dimensional spacetime (for a review see [1, 2]). Braneworlds naturally appear in the string/M theory context and provide a novel setting for discussing phenomenological and cosmological issues related to extra dimensions. The models introduced by Randall and Sundrum are particularly attractive [3, 4]. The corresponding spacetime contains two (RSI), respectively one (RSII), Ricci-flat brane(s) embedded on a five-dimensional Anti-de Sitter (AdS) bulk. It is assumed that all matter fields are confined on the branes and only the gravity propagates in the five dimensional bulk. More recently, alternatives to confining particles on the brane have been investigated and scenarios with additional bulk fields have been considered.

Although topological defects have been first analyzed in four-dimensional spacetime [5], they have been considered in the context of braneworld. In this scenario the defects live in a  $d$ -dimensional submanifold embedded in a  $(4+d)$ -dimensional Universe. The domain wall case, with a single extra dimension, has been considered in [6]. More recently the cosmic string case, with two additional extra dimensions, has been analyzed [7, 8]. For the case with three

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\*E-mail: emello@fisica.ufpb.br

†E-mail: saharayan@server.physdep.r.am

extra dimensions, the 't Hooft-Polyakov magnetic monopole has been numerically analyzed in [9, 10]. In Refs. [11]-[15] the corresponding analysis for the global monopole is presented. In particular, in [11] the authors have obtained the solution to the Einstein equations considering a general  $p$ -dimensional Minkowski brane worldsheet and a  $(m+1)$ -dimensional global monopole in the transverse extra dimensions with the core on the brane. The corresponding line element has the form

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu - dr^2 - \alpha^2 r^2 d\Omega_m^2, \quad (1)$$

where  $\eta_{\mu\nu}$  is the Minkowskian metric on the  $p$ -brane,  $\alpha^2 = 1 - \kappa^2 \eta_0^2 / (m-1)$  with  $\eta_0$  being the energy scale where the gauge symmetry of the global system is spontaneously broken,  $d\Omega_m^2$  is the line element on a  $m$ -sphere with unit radius.

In braneworld models the investigation of quantum effects is of considerable phenomenological interest, both in particle physics and in cosmology. The analysis of quantum effects produced by a massless scalar field propagating in the bulk described by  $m=2$  version of line-element (1) has been developed in [16] (for the investigation of the quantum vacuum effects in higher dimensional braneworld models with compact internal spaces see [17]-[20]). Continuing in this direction, our interest here is to investigate the quantum effects induced also by the conical structure of the brane. We will consider the cosmic string on the brane and the global monopole in transverse dimensions as idealized defects. The paper is organized as follows. In the next section we evaluate the heat kernel for a massless scalar field. In Section 3 we consider a special case where the angle deficit in the cosmic string subspace is an integer fraction of  $2\pi$  and evaluate the corresponding Euclidean Green function and the vacuum expectation values of the field square. The corresponding quantities for the general case of the angle deficit are evaluated in Section 4. We also consider the vacuum expectation value of the energy-momentum tensor.

## 2 Heat kernel

We consider a  $(D+1)$ -dimensional background spacetime having the structure of direct product of the cosmic string and global monopole subspaces with the line element

$$ds^2 = dt^2 - dp^2 - \rho^2 d\phi^2 - \sum_{i=1}^d dz_i^2 - dr^2 - \alpha^2 r^2 d\Omega_m^2, \quad (2)$$

where  $\phi \in [0, 2\pi/b]$ ,  $z_i \in (-\infty, \infty)$ , and  $D = d + m + 3$ . In the standard case with  $d=1$ , the parameter  $b$  is associated with the planar angle deficit and is related to the linear mass density of the string. In the special case  $b=1$ , line element (2) reduces to the interval (1) with  $p = d + 3$ . In this section we evaluate the heat kernel associated with a massless scalar field in the spacetime defined by (2) admitting an arbitrary curvature coupling parameter  $\xi$ . The corresponding Euclidean Green function obeys the second order differential equation

$$\left( \nabla_l \nabla^l + \xi R \right) G(x, x') = -\delta^{D+1}(x, x'), \quad (3)$$

with  $\nabla_l$  being the covariant derivative operator and  $R$  is the scalar curvature for the background spacetime. The most important special cases are minimally and conformally coupled scalar fields with  $\xi = 0$  and  $\xi = \xi_D \equiv (D-1)/4D$  respectively. For the geometry described by (2) one has  $R = m(m-1)(1/\alpha^2 - 1)/r^2$  for the points away from the string core. In order to evaluate the Green function we adopt the Schwinger-DeWitt formalism as shown below:

$$G(x, x') = \int_0^\infty ds K(x, x'; s), \quad (4)$$

where the heat kernel,  $K(x, x'; s)$ , can be expressed in terms of eigenfunctions of the operator  $\nabla_l \nabla^l + \xi R$  as follows:

$$K(x, x'; s) = \sum_{\sigma} \varphi_{\sigma}(x) \varphi_{\sigma}^*(x') \exp(-s\sigma^2), \quad (5)$$

$\sigma^2$  being the corresponding positively defined eigenvalue. Writing

$$\left( \nabla_l \nabla^l + \xi R \right) \varphi_{\sigma}(x) = -\sigma^2 \varphi_{\sigma}(x), \quad (6)$$

we obtain the complete set of normalized solutions of the above equation

$$\varphi_{\sigma}(x) = \left( \frac{q\lambda\alpha^{-mb}}{N(m_{\eta})} \right)^{\frac{1}{2}} \frac{e^{-i\omega\tau + inb\phi + i\mathbf{k}\cdot\mathbf{z}}}{(2\pi)^{n/2+1} r^{(m-1)/2}} J_{|n|b}(q\rho) J_{\nu_l}(\lambda r) Y(m_{\eta}; \vartheta, \Phi), \quad (7)$$

with  $\tau$  being the Euclidean time coordinate,  $\mathbf{z} = (z_1, \dots, z_d)$ ,  $k = |\mathbf{k}|$ ,  $n = 0, \pm 1, \dots$ , and

$$\sigma^2 = \omega^2 + k^2 + q^2 + \lambda^2. \quad (8)$$

The expression for  $N(m_{\eta})$  is given in [21] and will not be necessary in the following discussion. In (7),  $J_{\nu}(x)$  is the Bessel function,

$$\nu_l = \frac{1}{\alpha} \left[ \left( l + \frac{m-1}{2} \right)^2 + (1 - \alpha^2) m(m-1) (\xi - \xi_m) \right]^{\frac{1}{2}}, \quad (9)$$

the function  $Y(m_{\eta}; \vartheta, \Phi)$  is the hyperspherical harmonic of degree  $l$  [21] with  $m_{\eta} = (m_0 \equiv l, m_1, \dots, m_{m-1})$ , and  $m_1, m_2, \dots, m_{m-1}$  are integers such that

$$0 \leq m_{m-2} \leq m_{m-3} \leq \dots \leq m_1 \leq l, \quad -m_{m-2} \leq m_{m-1} \leq m_{m-2}. \quad (10)$$

So according to (5) the heat kernel is given by the expression

$$K(x, x'; s) = \int d\omega \int d^d k \int dq \int d\lambda \sum_n \sum_{m_{\eta}} \varphi_{\sigma}(x) \varphi_{\sigma}^*(x') e^{-s\sigma^2}.$$

By using the formula from [22] for the integrals involving the Bessel functions and the addition theorem for the spherical harmonics [21]

$$\sum_{m_{\eta}} \frac{Y(m_{\eta}; \vartheta, \Phi)}{N(m_{\eta})} Y^*(m_{\eta}; \vartheta', \Phi') = \frac{2l + m - 1}{(m-1)S_m} C_l^{(m-1)/2}(\cos \theta), \quad (11)$$

we obtain the following formula

$$\begin{aligned} K(x, x'; s) &= \frac{b(r r')^{(1-m)/2}}{(4\pi)^{p/2} \alpha^m} \frac{e^{-V/4s}}{s^{p/2+1}} \sum_{n=0}^{\infty} I_{nb} \left( \frac{\rho \rho'}{2s} \right) \cos(nb\Delta\phi) \\ &\quad \times \sum_{l=0}^{\infty} \frac{2l + m - 1}{(m-1)S_m} C_l^{(m-1)/2}(\cos \theta) I_{\nu_l} \left( \frac{r r'}{2s} \right), \end{aligned} \quad (12)$$

where  $I_{\nu}(x)$  is the modified Bessel function,  $C_p^q(x)$  is the Gegenbauer polynomial of degree  $p$  and order  $q$ ,

$$V = \Delta\tau^2 + \Delta\mathbf{z}^2 + \rho^2 + \rho'^2 + r^2 + r'^2,$$

and the prime means that the summand with  $n = 0$  should be taken with the weight  $1/2$ . In formula (12),  $\theta$  is the angle between the directions  $(\vartheta, \Phi)$  and  $(\vartheta', \Phi')$ , and  $S_m = 2\pi^{(m+1)/2}/\Gamma((m+1)/2)$  is the volume of the  $m$ -dimensional sphere. Note that  $m = 1$  corresponds to a cosmic string in transverse dimensions. In this case  $\nu_l = l/\alpha$  and the corresponding heat kernel is obtained from general formula (12) by taking into account the relation (see, for instance, [23])  $\lim_{\beta \rightarrow 0} l C_l^{\beta}(\cos \theta)/\beta = (2 - \delta_{l0}) \cos l\theta$  for the Gegenbauer polynomial.

### 3 Special case

Before to construct the Green function in the general case by using (4), we will consider a special case when the parameter  $b$  is an integer number. In order to provide the Green function let us go back, before to make the integration over  $q$ . Using the formula [22]

$$\sum_{n=-\infty}^{\infty} J_{|n|b}(q\rho)J_{|n|b}(q\rho')e^{inb\Delta\phi} = \frac{1}{b} \sum_{j=0}^{b-1} J_0(qv_j) , \quad (13)$$

with

$$v_j^2 = \rho^2 + \rho'^2 - 2\rho\rho' \cos(\Delta\phi - 2\pi j/b) , \quad (14)$$

we find

$$K(x, x'; s) = \frac{(rr')^{(1-m)/2}}{2(4\pi)^{p/2}\alpha^m} \sum_{j=0}^{b-1} \frac{e^{-V_j/4s}}{s^{p/2+1}} \sum_{l=0}^{\infty} \frac{2l+m-1}{(m-1)S_m} C_l^{(m-1)/2}(\cos\theta) I_{\nu_l}\left(\frac{rr'}{2s}\right) , \quad (15)$$

where

$$V_j = \Delta\tau^2 + \Delta\mathbf{z}^2 + r^2 + r'^2 + v_j^2 . \quad (16)$$

Now we are in position to obtain the Euclidean Green function by substituting (15) into (4). After the evaluation of the integral by using the formula from [22], our final result is:

$$\begin{aligned} G(x, x') &= -\frac{(-i)^d(2\pi)^{-d/2-2}}{\alpha^m(rr')^{(D-1)/2}} \sum_{j=0}^{b-1} \frac{1}{(\sinh u_j)^{d/2+1}} \times \\ &\quad \sum_{l=0}^{\infty} \frac{2l+m-1}{(m-1)S_m} C_l^{(m-1)/2}(\cos\theta) Q_{\nu_l-1/2}^{d/2+1}(\cosh u_j) , \end{aligned} \quad (17)$$

where

$$\cosh u_j = \frac{V_j}{2rr'} , \quad (18)$$

and  $Q_{\nu}^{\lambda}(x)$  is the associated Legendre function. By making use of the relation between the Legendre function and the hypergeometric function [23], formula (17) can also be written in the form

$$\begin{aligned} G(x, x') &= \frac{\alpha^{-m}}{(2\pi)^{\frac{p}{2}}(rr')^{\frac{D-1}{2}}} \sum_{l=0}^{\infty} \frac{2l+m-1}{(m-1)S_m} \frac{2^{-\nu_l-1}\Gamma(\mu_l)}{\Gamma(\nu_l+1)} C_l^{(m-1)/2}(\cos\theta) \\ &\quad \times \sum_{j=0}^{b-1} (\cosh u_j)^{-\mu_l} F\left(\frac{\mu_l+1}{2}, \frac{\mu_l}{2}; \nu_l+1; \frac{1}{\cosh^2 u_j}\right) , \end{aligned} \quad (19)$$

where we have introduced the notation

$$\mu_l = \nu_l + p/2. \quad (20)$$

From (19) we can observe that the  $j = 0$  component clearly presents a divergence at the coincidence limit. However, for the other components  $\cosh u_j$  will be always greater than unity and consequently the Legendre function assumes finite value. The  $j = 0$  term in formulae (17), (19) is the Green function for the geometry with  $b = 1$  when the cosmic string is absent. Formula (17) presents the Green function for the geometry with the cosmic string as an image sum of the  $b = 1$  Green functions.

The analysis of the vacuum expectation values (VEVs) associated with a massless scalar field in a spacetime defined by (2) in the absence of cosmic string has been presented in [16]. Here we are mainly interested in quantum effects induced by the presence of the string. To investigate these effects we introduce the subtracted Green function

$$G_{\text{sub}}(x, x') = G(x, x') - G(x, x')|_{b=1}. \quad (21)$$

As the presence of the string does not change the curvature for the background manifold for the points  $\rho \neq 0$ , the structure of the divergences in the coincidence limit is the same for both terms on the right hand side. Hence, for these points the function  $G_{\text{sub}}(x, x')$  is finite in the coincidence limit.

By using formula (17), the VEV of the field square is presented as the sum

$$\langle 0|\varphi^2|0\rangle = \langle \varphi^2 \rangle_{\text{m}} + \langle \varphi^2 \rangle_{\text{s}}, \quad (22)$$

where the first term on the right is the corresponding VEV in the case when the string is absent ( $b = 1$ ) and the second term is a new contribution induced by the cosmic string. The latter is directly obtained from the subtracted Green function in the coincidence limit:

$$\begin{aligned} \langle \varphi^2 \rangle_{\text{s}} &= -\frac{(-i)^d (2\pi)^{-d/2-2}}{\alpha^m S_m r^{D-1}} \sum_{j=1}^{b-1} \sum_{l=0}^{\infty} D_l \frac{Q_{\nu_l-1/2}^{d/2+1}(\cosh w_j)}{(\sinh w_j)^{d/2+1}} \\ &= \frac{(2\pi)^{-\frac{p}{2}} \alpha^{-m}}{2 S_m r^{D-1}} \sum_{l=0}^{\infty} \frac{D_l \Gamma(\mu_l)}{2^{\nu_l} \Gamma(\nu_l + 1)} \sum_{j=1}^{b-1} (\cosh w_j)^{-\mu_l} F\left(\frac{\mu_l + 1}{2}, \frac{\mu_l}{2}; \nu_l + 1; \frac{1}{\cosh^2 w_j}\right) \end{aligned} \quad (23)$$

where we have used the relation  $C_l^p(1) = \Gamma(l + 2p)/\Gamma(2p)l!$ . In formula (23) the factor

$$D_l = (2l + m - 1) \frac{\Gamma(l + m - 1)}{\Gamma(m) l!} \quad (24)$$

is the degeneracy of each angular mode with given  $l$  and

$$\cosh w_j = 1 + 2 \frac{\rho^2}{r^2} \sin^2(\pi j/b). \quad (25)$$

In figure 1 we have presented the VEV of the field square induced by the cosmic string for minimally (left panel) and conformally (right panel) coupled scalar fields as a function on  $\rho/r$  and  $\alpha$  in the model with  $d = 1$ ,  $m = 2$ ,  $b = 3$ . From this figure we see that the behavior of the field square for small values of the parameter  $\alpha$  depends essentially on the value of the curvature coupling parameter. For a minimally coupled scalar the VEV induced by the string increases with decreasing  $\alpha$ , whereas for a conformally coupled scalar this VEV vanishes in the limit  $\alpha \rightarrow 0$ . In the next section this behavior will be analytically derived from the corresponding formulae for the general case of the parameter  $b$ .

## 4 General Case

### 4.1 Green function

In order to evaluate the vacuum polarization effect induced by the cosmic string for general values of the parameter  $b$ , we consider the function which is obtained from the heat kernel (12) subtracting the corresponding heat kernel for the geometry without a cosmic string. The latter

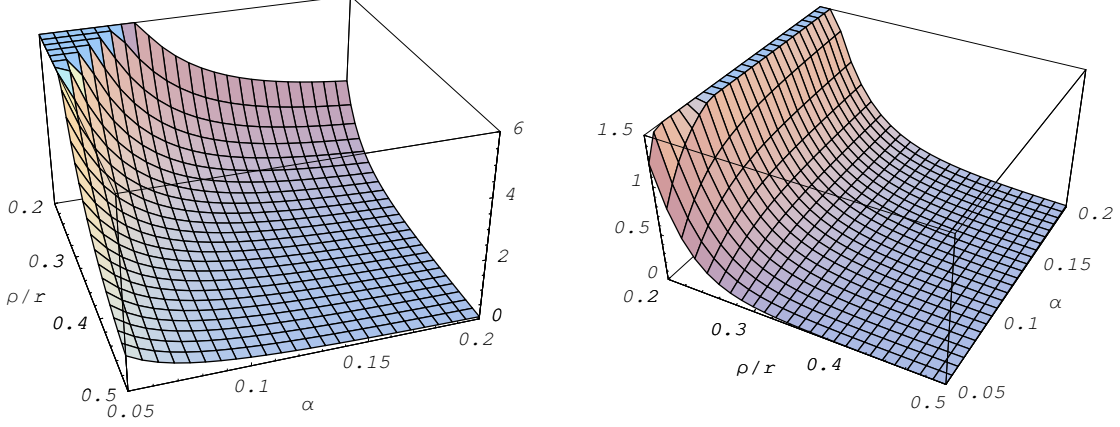


Figure 1: Part in the VEV of the field square induced by the cosmic string,  $r^{D-1}\langle\varphi^2\rangle_s$ , as a function on  $\rho/r$  and  $\alpha$  in the model with the parameters  $d = 1$ ,  $m = 2$ ,  $b = 3$ . The left and right panels correspond to minimally and conformally coupled scalar fields respectively.

is obtained from (12) substituting  $b = 1$ . By using the notation  $K_{\text{sub}}(x, x'; s)$  for the subtracted function, one finds

$$K_{\text{sub}}(x, x'; s) = \frac{(rr')^{(1-m)/2}}{(4\pi)^{p/2}\alpha^m} \frac{e^{-V/4s}}{s^{p/2+1}} \sum_{l=0}^{\infty} \frac{2l+m-1}{(m-1)S_m} C_l^{(m-1)/2}(\cos\theta) I_{\nu_l}\left(\frac{rr'}{2s}\right) \times \sum_{n=0}^{\infty} \left[ b I_{nb}\left(\frac{\rho\rho'}{2s}\right) \cos(nb\Delta\phi) - I_n\left(\frac{\rho\rho'}{2s}\right) \cos(n\Delta\phi) \right]. \quad (26)$$

To provide a more convenient expression for the subtracted heat kernel, we apply the Abel-Plana formula (see, for instance, [24, 25]) for the summation over  $n$ :

$$\sum_{n=0}^{\infty} F(n) = \int_0^{\infty} du F(u) + i \int_0^{\infty} du \frac{F(iu) - F(-iu)}{e^{2\pi u} - 1}.$$

In this formula we take  $F(u) = I_{uw}(z) \cos(uw\Delta\phi)$  with  $z = \rho\rho'/2s$  and  $w = b, 1$  for the first and second terms in the square brackets in (26), respectively. Now we can see that in the evaluation of the difference the terms coming from the first integral on the right of Abel-Plana formula cancel out and one obtains

$$\sum_{n=0}^{\infty} [b I_{nb}(z) \cos(nb\Delta\phi) - I_n(z) \cos(n\Delta\phi)] = \frac{2}{\pi} \int_0^{\infty} du \cosh(u\Delta\phi) g(b, u) K_{iu}(z), \quad (27)$$

where  $K_{\nu}(z)$  is the MacDonald function and we have introduced the notation

$$g(b, u) = \sinh(\pi u) \left( \frac{1}{e^{2\pi u/b} - 1} - \frac{1}{e^{2\pi u} - 1} \right). \quad (28)$$

The respective subtracted Green function becomes:

$$G_{\text{sub}}(x, x') = \frac{(rr')^{(1-D)/2}}{2^{p/2-1}\pi^{p/2+1}\alpha^m} \sum_{l=0}^{\infty} \frac{2l+m-1}{(m-1)S_m} C_l^{(m-1)/2}(\cos\theta) \times \int_0^{\infty} du g(b, u) \cosh(u\Delta\phi) \int_0^{\infty} dv v^{p/2-1} e^{-Vv/2rr'} I_{\nu_l}(v) K_{iu}\left(\frac{\rho\rho'}{rr'}v\right). \quad (29)$$

For the points outside the cores of the topological defects, the expression on the right of this formula is finite in the coincidence limit and can be directly used for the evaluation of the vacuum expectation values of the field square and the energy-momentum tensor.

## 4.2 VEVs for the field square and energy-momentum tensor

In formula (29), taking the coincidence limit for the part of the VEV of the field square induced by the cosmic string one finds

$$\begin{aligned} \langle \varphi^2 \rangle_s &= \frac{2^{1-p/2} r^{1-D}}{\pi^{p/2+1} \alpha^m S_m} \sum_{l=0}^{\infty} D_l \int_0^{\infty} du g(b, u) \\ &\times \int_0^{\infty} dv v^{p/2-1} e^{-(1+y)v} I_{\nu_l}(v) K_{iu}(yv), \end{aligned} \quad (30)$$

where  $y = \rho^2/r^2$ . For the case  $\alpha = 1$  corresponding to the absence of the global monopole one has  $\nu_l = l + (m-1)/2$  and the summation over  $l$  can be done explicitly by using the formula

$$\sum_{l=0}^{\infty} D_l I_{l+\nu}(v) = \frac{e^v}{v \Gamma(\nu)} \left( \frac{v}{2} \right)^{\nu}. \quad (31)$$

(This formula is obtained from a more general addition theorem given in [22].) After the evaluation of the  $v$ -integral with the help of formula from [22], one finds

$$\langle \varphi^2 \rangle_s|_{\alpha=1} = \frac{(2\rho)^{1-D}}{\pi^{\frac{D}{2}+1} \Gamma(\frac{D}{2})} \int_0^{\infty} du g(b, u) \left| \Gamma\left(\frac{D-1}{2} + iu\right) \right|^2. \quad (32)$$

For odd values  $D$  the modulus of the gamma function in this formula is expressed via the elementary functions and the integral is explicitly evaluated. In particular, for  $D = 3$  one has  $|\Gamma(1 + iu)|^2 = \pi u / \sinh(\pi u)$  and from (32) we obtain the well-known result [26, 27]. The result for the case  $D = 5$  with the recurrence relations for the evaluation of the higher odd values are given in [28].

Now we consider the behavior of the string induced VEV  $\langle \varphi^2 \rangle_s$  in the asymptotic regions for the parameter  $y$ . For large values of  $y$ , introducing in (30) a new integration variable  $z = vy$  and expanding the integrand over  $1/y$ , we see that the dominant contribution comes from the  $l = 0$  term. To the leading order one has  $\langle \varphi^2 \rangle_s \sim r^{2\nu_0+1-m}/\rho^{2\nu_0+p}$ . In particular, the VEV induced by the string diverges on the core of the global monopole for  $2\nu_0 < m-1$ , is finite for  $2\nu_0 = m-1$  corresponding to  $\alpha = 1$ , and vanishes for  $2\nu_0 > m-1$ . For a fixed value  $r$  and at large distances from the string core  $\langle \varphi^2 \rangle_s$  vanishes as  $1/\rho^{2\nu_0+p}$ . As the VEV of the field square given by (30) diverges on the string core corresponding to  $y = 0$ , in the limit  $y \ll 1$  the main contribution into the sum over  $l$  comes from large values  $l$  and we use the uniform asymptotic expansion for the function  $I_{\nu_l}(v)$ . As the next step we replace the summation over  $l$  by the integration,  $\sum_l D_l f(\nu_l) \rightarrow (2\alpha^m/\Gamma(m)) \int_0^{\infty} dx x^{m-1} f(x)$ , and change the order of the integrations. Further introducing in the integral over  $x$  a new integration variable  $t = x/v$ , the integral over  $t$  is estimated by the Laplace method. After the evaluation of the remained integral over  $v$  using formula from [22], it can be seen that to the leading order  $\langle \varphi^2 \rangle_s$  coincides with the corresponding result for  $\alpha = 1$  case given by (32). This means that near the cosmic string the most relevant contribution to the VEV comes from the string itself.

For small values of the parameter  $\alpha$ , corresponding to strong gravitational field, and for a non-minimally coupled scalar field one has  $\nu_l \gg 1$  for all values  $l$ . Replacing the function  $I_{\nu_l}(v)$  by the corresponding uniform asymptotic expansion and noting that the dominant contribution

into the integral over  $v$  comes from the values  $v \sim \nu_l$ , we can estimate this integral by the Laplace method. In this way it can be seen that the main contribution comes from  $l = 0$  term and this contribution is exponentially suppressed. In the same limit,  $\alpha \ll 1$ , and for a minimally coupled scalar field one has  $\nu_l \gg 1$  for  $l \neq 0$  terms and their contribution is exponentially suppressed. For  $l = 0$  term one has  $\nu_0 = (m-1)/2$  and the corresponding contribution to the string induced VEV of the field square behaves as  $\langle \varphi^2 \rangle_s \sim \alpha^m$ . As we see, in this limit the behavior of the VEV as a function on  $\alpha$  is essentially different for minimally and non-minimally coupled scalar fields. This is also seen from figure 1. The similar feature takes place for the VEVs induced in the global monopole bulk by the presence of boundaries [29]-[33]. The suppression for the case of a non-minimally coupled scalar field can be understood if we note that for small values  $\alpha$  one has  $R \approx 1/\alpha^2 r^2$  and the term  $\zeta R$  in the field equation acts like the mass squared term.

Now we turn to the evaluation of the VEV for the energy-momentum tensor. As in the case of the field square, this VEV is presented in the form of the sum of the purely global monopole and string induced parts:

$$\langle 0|T_{ik}|0\rangle = \langle T_{ik} \rangle_m + \langle T_{ik} \rangle_s. \quad (33)$$

The string induced part is obtained by using the formula

$$\langle T_{ik} \rangle_s = \lim_{x' \rightarrow x} \partial_i \partial'_k G_{\text{sub}}(x, x') + \left[ \left( \xi - \frac{1}{4} \right) g_{ik} \nabla_l \nabla^l - \xi \nabla_i \nabla_k - \xi R_{ik} \right] \langle \varphi^2 \rangle_s, \quad (34)$$

where  $R_{ik}$  is the Ricci tensor for the background spacetime,  $i, k = 0$  components correspond to the standard time coordinate  $t$ , and the values of the indices  $i = 1, 2, \dots, D$  correspond to the coordinates  $(\rho, \phi, z_1, \dots, z_d, r, \theta_1, \dots, \theta_{m-1}, \Phi)$  respectively. Note that in the first term on the right of this formula, before the differentiation, the rotation on the time coordinate should be made in the formula for  $G_{\text{sub}}(x, x')$ . From the symmetry of the problem it follows that the vacuum stresses  $\langle T_i^i \rangle_s$ ,  $i = p+1, \dots, D$ , are isotropic and one has the relations (no summation over  $i$ )

$$\langle T_0^0 \rangle_s = \langle T_i^i \rangle_s, \quad i = 3, \dots, p, \quad (35)$$

between the corresponding energy density and vacuum stresses along the string axis. From the continuity equation  $\nabla_k \langle T_i^k \rangle_s = 0$  we obtain the following relations for the components of the energy-momentum tensor (no summation over  $i$ ):

$$\langle T_2^2 \rangle_s = \frac{\partial}{\partial \rho} (\rho \langle T_1^1 \rangle_s), \quad \langle T_i^i \rangle_s = \left( 1 + \frac{r}{m} \frac{\partial}{\partial r} \right) \langle T_p^p \rangle_s, \quad (36)$$

with  $i = p+1, \dots, D$ . It is useful also to take into account the trace relation

$$\langle T_i^i \rangle_s = D(\xi - \xi_D) \nabla_l \nabla^l \langle \varphi^2 \rangle_s. \quad (37)$$

For the expression appearing on the right hand side of formula (37) we have

$$\begin{aligned} \nabla_l \nabla^l \langle \varphi^2 \rangle_s &= -\frac{r^{-D-1} \alpha^{-m}}{2^{p/2-2} \pi^{p/2+1} S_m} \sum_{l=0}^{\infty} D_l \int_0^{\infty} du g(b, u) \int_0^{\infty} dv v^{p/2-1} e^{-(1+y)v} \\ &\times \{ 2 [2v^2(1+y) - 2v + \nu_l^2 - u^2/y] I_{\nu_l}(v) K_{iu}(yv) \\ &- v(4v + m - 1) I'_{\nu_l}(v) K_{iu}(yv) - v(4yv - m + 1) I_{\nu_l}(v) K'_{iu}(yv) \}. \end{aligned} \quad (38)$$

By using (34), for the remained components of the energy-momentum tensor we find the formulae (no summation over  $i$ )

$$\begin{aligned} \langle T_i^i \rangle_s &= -\frac{r^{-1-D} \alpha^{-m}}{2^{p/2-1} \pi^{p/2+1} S_m} \sum_{l=0}^{\infty} D_l \int_0^{\infty} du g(b, u) \int_0^{\infty} dv v^{p/2-1} \\ &\times e^{-(1+y)v} F^{(i)}(y, u, v) + \left( \xi - \frac{1}{4} \right) \nabla_l \nabla^l \langle \varphi^2 \rangle_s, \end{aligned} \quad (39)$$



where  $y$  is defined after formula (30) and we have introduced notations

$$F^{(0)}(y, u, v) = v I_{\nu_l}(v) K_{iu}(yv), \quad (40)$$

$$F^{(1)}(y, u, v) = \frac{1}{y^2} I_{\nu_l}(v) \{ -yv [2yv(1 - 4\xi) - 2\xi] K'_{iu}(yv) + [(2y^2v^2 - u^2)(1 - 4\xi) + 2\xi yv] K_{iu}(yv) \}, \quad (41)$$

$$F^{(p)}(y, u, v) = K_{iu}(yv) \{ 2v [(\beta + v)(4\xi - 1) + \xi] I'_{\nu_l}(v) + [(2v^2 + \nu_l^2 + 2\beta v + \beta^2)(1 - 4\xi) + 2\xi(v - \beta)] I_{\nu_l}(v) \}, \quad (42)$$

with  $\beta = (m - 1)/2$ . The consideration of the asymptotic cases of the general formulae for the components of the energy-momentum tensor is similar to that for the field square. For large values of  $y$  the main contribution comes from the  $l = 0$  term and these components behave as  $r^{-D-1}(\rho/r)^{p+2\nu_0}$ . In the limit  $y \ll 1$  corresponding to the points near the core of the cosmic string, to the leading order the energy-momentum tensor coincides with the corresponding quantity for the  $\alpha = 1$  case. In particular, for the energy density one has

$$\begin{aligned} \langle T_0^0 \rangle_s &\approx \langle T_0^0 \rangle_s|_{\alpha=1} = -\frac{2^{1-D}\rho^{-1-D}}{\pi^{\frac{D}{2}+1}\Gamma(\frac{D}{2})} \int_0^\infty du g(b, u) \\ &\times \left| \Gamma\left(\frac{D-1}{2} + iu\right) \right|^2 \left[ (D-1)^2(\xi - \xi_D) + \frac{u^2}{D} \right]. \end{aligned} \quad (43)$$

As a special case, for  $D = 3$  we obtain the result given in [34]. For small values of the parameter  $\alpha$  the dominant contribution to the vacuum energy-momentum tensor induced by the cosmic string comes from the term with  $l = 0$  and this tensor is exponentially suppressed for the non-minimally coupled scalar and behaves like  $1/\alpha^m$  for a minimally coupled scalar.

## 5 Conclusion

In this paper we have investigated quantum vacuum effects for a massless scalar field induced by a composite topological defect. Specifically we have considered the spacetime being a direct product of the cosmic string and global monopole geometries. The corresponding heat kernel is constructed and on the base of this the Green function is evaluated. The complete Green function is composed by two terms: (i) the first one being singular at the coincidence limit; this term contains information only on the global monopole defect, and (ii) a regular term which contains information about the presence of both topological defects and vanishes in the absence of the cosmic string ( $b = 1$ ). First we consider the case when the parameter  $b$  describing the planar angle deficit in the cosmic string geometry is an integer. In this case the Green function is presented as an image sum of the Green functions corresponding to the geometry which has a structure of a  $p$ -dimensional Minkowskian brane with a global monopole in transverse dimensions. The vacuum polarization in the latter geometry is considered in [16] and our main interest here are the effects induced by the cosmic string. For the points away from the cores of the topological defects the corresponding part in the Green function is finite in the coincidence limit and can be directly used for the evaluation of the vacuum expectation values of the field square and the energy-momentum tensor. In the special case with integer values of the parameter  $b$  the string induced part in the VEV of the field square is given by formula (23). The general case of the planar angle deficit is considered in Section 4. By using the Abel-Plana formula for the summation over the azimuthal quantum number, we have explicitly subtracted from the Green function the part corresponding to the geometry when the cosmic string is absent.

The corresponding VEV of the field square is obtained in the coincidence limit and is given by formula (30). We have investigated the general formula in various limiting cases for the parameters of the model. In particular, we have shown that for small values of the parameter  $\alpha$  corresponding to strong gravitational fields, the behavior of the field square is essentially different for minimally and non-minimally coupled fields. For points near the string core, in the leading order, the vacuum densities coincide with those for the geometry when the global monopole is absent corresponding to  $\alpha = 1$ . We have also evaluated the the VEV of the energy-momentum tensor induced by the string. The corresponding independent components are given by formulae (39)-(42). Other components can be obtained by using the continuity equation from relations (36).

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